Influence of variable β

The accuracy of the constant β assumption is investigated[3] by calculating the flux to the body with constant β and comparing it with the current results (variable β). The results are presented for ground state radiation only in Fig. 2 which shows the absolute value of the difference between the constant β_o flux and variable β_o flux divided by the variable β_o flux as a function of the shock-layer thickness for Mach 30 and Mach 50. When the shock-layer is optically thin, the variable β_o flux is greater than the constant β_o flux. As the shock layer becomes optically thick, the constant β_o flux becomes greater than the variable β_o flux. The constant β_o flux and variable β_o flux become equal near an optical thickness of unity.

When τ_o is small, the flux is proportional to τ_o; consequently, the variable β_o flux is greater than the constant β_o flux because the variable β_o shock layers have larger optical thicknesses. When τ_o becomes large, attenuation becomes important. Thus, the variable β_o flux is less than the constant β_o flux because the optical thickness of the variable β_o shock layers is greater. One can conclude from Fig. 2 that for shock layer thicknesses of less than 0.1 cm, the error in the constant β_o approximation is less than one percent. As the thickness increases and θ_o decreases, the error in the constant β_o approximation increases.

Influence of shock layer thickness

Figure 3 gives the total flux to the body from the ground state and the excited state as a function of the shock layer thickness for Mach 30 and Mach 50, respectively. The ground state flux is larger than the excited state flux for small values of the shock layer thickness L. As L increases, the ground state flux increases and reaches its maximum and then falls off rapidly because of self-absorption and the temperature profile. It quickly becomes negligible with respect to the excited state flux. The exchange between the ground state and the excited state as the dominant source of radiation appears as a rapid change of slope near $L = 0.1$ cm. As L continues to increase, the excited state flux reaches its maximum and begins to fall off due to the same processes.

Figure 3 shows that the maximum ground state flux for the constant β assumption occurs at slightly larger plasma thicknesses than it does for variable β. For the excited state flux the maximum for the constant β assumption occurs at slightly smaller plasma thicknesses than it does for the variable β case.

For large L the constant β_1 assumption overestimates self-absorption. The local temperature determines the excited state population; therefore, the variable β_1 plasma has a cool region where its excited state is optically thin. Hence, more of the radiation emitted near the shock wave reaches the body for the variable β_1 case, than for the constant β_1 case.

At small shock layer thicknesses, the assumption of constant β gives a better approximation to the total flux than it does for the ground and excited state fluxes individually because they vary in opposite directions cancelling out the error. As L becomes large, the flux becomes almost entirely due to the excited state so that the assumption of constant β introduces errors of the same magnitude as it does for the excited state alone. One can conclude from this analysis that the constant β approximation gives results that show the same trends and roughly the same magnitudes as the variable β calculations.

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Measurements of Skin Friction on the Wall of a Hypersonic Nozzle

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Nomenclature

C_f	= local skin-friction coefficient, $2\tau_w/\rho_e u_e^2$
M	= Mach number
p	= pressure
R_e	= momentum thickness Reynolds number, $\theta\rho_e u_e/\mu_e$
T	= temperature
u	= velocity
x	= axial distance from nozzle throat
θ	= momentum thickness
τ	= shear stress
ρ	= density
μ	= viscosity

Subscripts

aw	= adiabatic temperature
c	= calculated
e	= boundary-layer edge
w	= wall
0	= settling chamber

Introduction

COMPARISONS of mean profile data in turbulent boundary-layers on the walls of hypersonic nozzles have been made for a wide range of Mach numbers.[1] In general, boundary-layers on nozzle walls are subjected to streamwise pressure and temperature gradients during their initial development. As a result of these gradients, a quadratic variation of total temperature with velocity occurs in the outer part of the boundary-layer.[2] The quadratic variation is typical of cold-wall nozzle boundary layers while a nearly linear variation occurs in most flat plate boundary-layers.[3] Comparisons[4–6] of nozzle-wall and flat-plate skin-friction data with commonly used prediction methods have indicated that surface shear-stress and heating may not be strongly influenced by upstream history. Recent reviews by Beckwith[7] and Cary[8] suggest a need for accurate skin-friction and heat-transfer data at hypersonic Mach numbers and cold-wall conditions in order to test the validity of various prediction methods for such conditions.

To help satisfy this need, direct measurements of skin friction have been obtained on the wall of a Mach 19.8 nozzle. Mean profile data in the turbulent boundary layer on the wall of this nozzle have been published.[1] The new direct measurements of

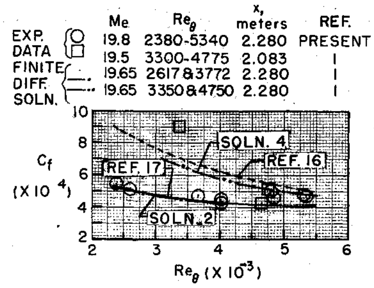

Fig. 1 Skin-friction variation with Re_θ, $T_w/T_o \simeq 0.18$.

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Table 1 Test conditions and experimental data, $X = 2.28$

M_e	T_o (K)	p_o (N/cm^2)	T_w (K)	T_e/T_o	T_w/T_{aw}	τ_w/p_o $(x\ 10^8)$	C_f $(x\ 10^4)$	R_e	θ (cm)
19.56	1658	2200	307	0.01290	0.2082	3.599	5.500	2380	0.1835
19.57	1632	2360	297	0.01289	0.2073	3.314	5.076	2605	0.1831
19.64	1592	3570	307	0.01280	0.2182	2.715	4.26	4015	0.1808
19.66	1655	3610	305	0.01277	0.2089	3.045	4.78	3660	0.1809
19.65	1632	3700	318	0.01279	0.2206	2.862	4.50	4030	0.1807
19.74	1600	4510	312	0.01267	0.2215	2.900	4.65	4840	0.1759
19.76	1632	4660	322	0.01264	0.2237	2.038	5.02	4790	0.1745
20.02	1600	5760	332	0.01235	0.2341	2.795	4.766	5340	0.1542
20.08	1590	5960	320	0.01233	0.2265	2.745	4.810	5310	0.1490

skin friction and values deduced from the profile data are compared with predictions from a finite difference theory[2] and also with several "flat-plate" prediction methods (termed "correlation techniques" in Ref. 7) for skin friction.

Test Facility and Conditions

The surface shear stress data were obtained on the wall of the Langley hypersonic nitrogen tunnel[1] (axisymmetric nozzle) 2.28 downstream of the throat for a momentum thickness Reynolds number range from 2380 to 5340, wall-to-stagnation temperature ratios of 0.187–0.203, and a nominal freestream Mach number of 19.8.

Instrumentation

The null-type skin-friction balance had the exposed surface contoured to match the local nozzle wall surface. A description of a similar balance is available.[9] The present balance has a floating element diameter of 1.27 cm with a peripheral gap width of 0.0076 cm. The measured wall temperature in the vicinity of the water-cooled balance never exceeded 340°K during any test; thereby eliminating errors due to temperature sensitivity. The balance had a full scale range of 0.15 g/cm^2 with a linear calibration repeatable to an accuracy of better than 1%.

Experimental Results and Comparison with Theory

Test conditions and experimental data at $x = 2.28$ m are shown in Table 1. Local values of C_f were obtained from measured values of τ_w and local values of $\rho_e u_e^2$ computed by assuming isentropic expansion from the tabulated settling chamber conditions to the listed values of M_e. The values of C_f are estimated to be accurate to within about 10% based upon an error analysis of these quantities. Boundary-layer surveys at $x = 2.28$ m were not made, hence values of θ (Table 1) were obtained by extrapolation of earlier data at $x = 2.083$ m by use of the momentum integral equation.

The present values of skin friction and previous values[1] obtained from slopes of Mach number gradient at the wall are plotted in Fig. 1 against R_{e_θ}. The measured data show little variation with R_{e_θ} and agree with previous data[1] except for the value at $R_{e_\theta} \simeq 3300$. The disagreement at this R_{e_θ} is believed to be caused by the large pitot probe errors[1] at the lower tunnel pressure.

The measured C_f values are compared in Table 2 with eight flat-plate prediction methods commonly used for compressible flow skin friction.[10–16] The present deviation of each data point from the flat-plate type predictions is shown in the table listed in the same order as in Table 1. The method of Moore[17] is the only theory that utilizes a quadratic relation between total temperature and velocity. All other theories shown in Table 2 use a linear variation. The relationships presented in Ref. 18 were used to transform values of C_f and R_{e_θ} for the predictions of Spalding and Chi.[11]

The results (Table 2) indicate that all of the flat-plate theories, except Coles,[16] Moore,[17] and Van Driest II[13] underpredict the measured values from about 1.6% to over 100%. The Van Driest II,[13] Coles,[16] and Moore[17] theory gives the best over all prediction of the experimental data for the momentum Reynolds number range obtained in the tests. Prediction curves are shown on Fig. 1 from the theories of Coles[16] and Moore[17] using values of T_w/T_{aw} and M_e of Table 1. While some of the discrepancies between predictions and data are probably due to experimental errors, it is apparent that surface shear stress under the turbulent boundary-layer of a cold-wall nozzle rapidly adjusts to local gradients and may be satisfactorily predicted by the conventional flat-plate methods of Van Driest II,[13] Moore,[17] and Coles.[16]

Calculated values from a finite difference solution are also shown in Fig. 1 for comparison. Values of input parameters used in these solutions were the same as those for solutions numbered 2 and 4 in Ref. 1. The value of dM/dx at $x = 2.28$ m was 0.068 per cm. The differences in the predicted levels of C_f from the two solutions are due to different eddy viscosity models described in Ref. 1.

Table 2 Percent deviation of data from predictions $C_f - C_{fc}/C_{fc}$

Eckert (Ref. 10)	Spalding-Chi (Ref. 11)	Sommers-Short (Ref. 12)	Van Driest II (Ref. 13)	Harkness (Ref. 14)	Baronti-Libby (Ref. 15)	Coles (Ref. 16)	Moore (Ref. 17)
57.9	106.1	45.8	19.82	35.5	33.4	−38.2	4.8
49.2	94.4	37.7	13.2	26.0	27.6	−40.0	− 1.7
44.9	80.9	33.6	8.5	23.1	30.9	−31.8	− 6.3
57.7	99.0	45.3	18.8	32.8	40.9	−27.6	1.3
54.4	91.4	42.3	15.4	31.4	39.5	−26.3	0.02
68.2	107.3	55.0	25.4	43.0	54.9	−15.7	7.9
19.5	46.0	10.0	−11.1	1.6	9.9	−39.9	−23.3
83.9	112.2	69.3	35.7	57.3	70.3	− 5.5	18.4
84.2	124.7	69.6	36.4	56.7	70.4	− 5.7	17.5

Predictions from the finite difference solutions are no better than those from the simple flat-plate type methods of Van Driest II,[13] Moore,[17] and Coles.[16] Of course, the use of these latter methods depends on knowing the values of R_{e_θ} which must be obtained by some other means. If possible, experimental profile data should be used for this purpose; however, integral methods may be used since predicted values of $R_{e\theta}$ at the nozzle exit depend mainly on the pressure gradient history rather than the skin-friction law.

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Aerodynamic Characteristics of Two-Dimensional Waverider Configurations

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MANNED shuttle flights into near space will necessitate increased maneuverability and performance of the vehicle that re-enters the upper atmosphere. The overriding aerodynamic parameter in such operations has long been known to be the hypersonic ligt-to-drag ratio (L/D).[1–3]

At high re-entry velocities, marked decreases in entry corridors, landing sites, and mid-course corrections occur. It becomes desirable therefore, to investigate vehicles whose L/D ratios provide a mission with more operational flexibility. Consideration of all these factors have led to the study of advanced Earth entry vehicles of relatively simple shapes having L/D's greater than Apollo's. One such study has led to the Waverider concept. This concept grew out of the need to consider new vehicle designs with high L/D's but whose generated flowfields were simple enough to be mathematically tractable. Since at hypersonic velocities the pressure distribution on the lower surface dominates the lifting characteristics of the wing, designs of flat-top wings were the first to be investigated. T. Nonweiler[4] was the first to consider the concept of inscribing a three-dimensional body within a two-dimensional body-shock configuration. Nonweiler's method of construction consisted of a two-dimensional wedge whose upper surface was colinear to the freestream velocity vector and whose lower surface was inclined at some angle θ_w with respect to the freestream velocity vector. This wedge generates a shock whose angle could easily be calculated from inviscid shock theory. If one now constructs a pair of swept leading edges in the shock plane from the vertex of the wedge, a three-dimensional body is formed (Fig. 1). This body, at the design freestream Mach number, will carry beneath it, in the plane of its swept leading edges, a planar trangular shock. Since the flowfield between the triangular shock and the body is the same as that between the original two-dimensional wedge and its shock, the flow characteristics of this region are immediately known from simple wedge theory. Once the flowfield is known, the lift and drag coefficients of the three-dimensional body due to pressure are calculable, keeping in mind the initial premise; that is, the lower surface dominates the aerodynamic characteristics of the body.

Two disadvantages become immediately apparent: 1) due to its construction, the Waverider is a blunt-based body, and 2) the Waverider's unusual shape (volume distribution and negative dihedral) makes it questionable from a practical engineering design viewpoint. However, both these disadvantages, while contributory to its over-all value as a re-entry vehicle, will not be discussed in this Note nor their subsequent consequences considered in the present analytical treatment.

Previous theoretical treatments of this problem and extensions to axisymmetric configurations are described in Ref. 5–8. Also presented therein are experimental investigations of Waverider designs in both the supersonic and hypersonic regimes. Restricting the present discussion to two-dimensional waveriders, K. Kipke[8] describes the current procedures of analyzing in-

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